

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

TMA1301 – COMPUTATIONAL METHODS

(All sections / Groups)

4 MARCH 2020
9.00am – 11.00am
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **4 pages** with **3 questions** only.
2. Attempt **ALL THREE** questions. The distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided and start each solution of a question on a **new page**.
4. Show all steps.

QUESTION 1 (5 MARKS)

Given the function $f(x) = \frac{\sin x}{x} - \cos x$.

- (a) Rewrite the given $f(x)$ to avoid loss of significance by using the first two nonzero terms in Taylor series expansion. [2.5 marks]

[HINT: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$; $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$; $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$]

- (b) Calculate the approximated value of $f(0.25)$ by using **SIX DIGIT arithmetic** with rounding. [1 mark]
- (c) The actual value of $f(0.25)$ is 0.02070. Find the relative error from the result obtained in (b). Correct your answer into **FIVE decimal places**. [1.5 mark]

Continued...

QUESTION 2 (15 MARKS)

(a) Consider the function $f(x) = x^3 - 5x^2 + 7x - 3$.

(i) Find $f'(x)$ and $f''(x)$.

[1 mark]

(ii) Using Newton's method, find a root of $f(x)$ starting with $p_0 = 0.9$ by completing the following table. Use **FOUR decimal places** for all workings.

[3 marks]

Iteration, n	p_n	$f(p_n)$	$f'(p_n)$
0			
1			
2			

(iii) Given the root of $f(x)$ is $r = 1$, is the convergence linear or quadratic as Newton's method is used to find the root?

[1.5 marks]

(iv) Based on the answer obtained in (iii), find the convergence rate. Hence express the error e_{n+1} in terms of the previous error e_n .

[1.5 marks]

(b) Consider the definite integral $\int_2^3 \frac{1}{x} dx$. Use **FOUR decimal places** for all workings.

(i) Find the actual value for the definite integral.

[1.5 marks]

(ii) Find the number of subintervals if Composite Trapezoidal Rule is used to approximate the definite integral with an error of at most 2×10^{-3} .

[Hint: error formula : $-\frac{b-a}{12}h^2 f''(x)$]

[4.5 marks]

(iii) With the number of subintervals obtained in (ii), find the approximated value for the definite integral using the Composite Trapezoidal Rule.

[2 marks]

Continued...

QUESTION 3 (20 MARKS)

- (a) Consider the following linear system:

$$\begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

- (i) Use row reduction technique to find an upper triangular **U** and lower triangular **L** in the **LU factorization** of the given linear system. [2.5 marks]
- (ii) Then find the values of x_1 , x_2 and x_3 by using the **L** and **U** obtained from (i). [3 marks]

- (b) Given the following system of equations:

$$7w - 2x + y + 2z = 3$$

$$2w + 8x + 3y + z = -2$$

$$-w + 5y + 2z = 5$$

$$2x - y + 4z = 4$$

- (i) Construct the equations for w , x , y and z of the above linear system using Jacobi Method. [2 marks]
- (ii) Copy the following table into your Answer Booklet. Complete it by computing one iteration of the Jacobi Method for the constructed equations in (i) starting with $[w, x, y, z] = [0, -1, 1, 1]$. Write your answers correct to **FOUR decimal places**. [2 marks]

n	w	x	y	z
0	0	-1	1	1
1				

- (c) Determine whether $\lambda = -7$ is the eigenvalue of matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. [1.5 marks]

- (d) Consider the following data:

x	$x_0 = 1$	$x_1 = 2$	$x_2 = 3$
$f(x)$	3	5	4

- (i) Construct the Lagrange Coefficients $L_0(x)$, $L_1(x)$ and $L_2(x)$. [1.5 marks]
- (ii) Find the second interpolating polynomial $P_2(x)$ using the results from (i). [1 mark]
- (iii) Approximate the value of $f(1.5)$ from the $P_2(x)$ obtained in (ii). [0.5 mark]

Continued...

- (e) The owner of a motor workshop keeps track on number of cars repaired and the revenue obtained. Based on a sample data, he has the following information:

Number of cars repaired	25	10	15	12	16	24	20	18	10	20	20	18
Revenue(RM'000)	8	3.5	6.5	6	7.5	9	7.8	8	6.8	7	6.5	7

- (i) Copy the following table into your Answer Booklet and complete it.

x	y	x^2	xy
25	8		
10	3.5		
15	6.5		
12	6		
16	7.5		
24	9		
20	7.8		
18	8		
10	6.8		
20	7		
20	6.5		
18	7		
$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum xy =$

[2 marks]

- (ii) From (i), find the equation of the best fit linear line $y = a + bx$ that models the data by using the least squares method. Round your answers to **FOUR decimal places**.

$$\text{[Hint: } a = \frac{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i - \sum_{i=1}^n x_i y_i \sum_{i=1}^n x_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}, \quad b = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}]$$

[3 marks]

- (iii) From (ii), estimate the revenue (in RM) when 10 cars are repaired.

[1 mark]

End of Page

